Entanglement and Mixed ness of Locally Cloned Non - Maximal W - State

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Abstract

In this work we describe a protocol by which two of three parties generate two bipartite entangled state among themselves without involving third party, from a non maximal W state or W - type state $|X\rangle = \alpha|001\rangle_{123} + \beta|010\rangle_{123} + \gamma|100\rangle_{123}, \alpha^2 + \beta^2 + \gamma^2 = 1$ shared by three distant partners. Also we have considered the case $\beta = \gamma$, to obtain a range for α^2, for which the local output states are separable and non local output states are inseparable. We also find out the dependence of the mixed ness of inseparable states with their amount of inseparability, for that range of α^2.

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1 Introduction:

For decades , quantum entanglement have been the focus of much of the work in the foundation of quantum mechanics. In particular, it's genesis comes with the concepts of

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non - separability, the violation of Bell Inequalities and EPR paradox. Creation and operation with entangled states are essential for quantum information application.Some of the applications are quantum teleportation [1], quantum dense coding [2], quantum error correction [3], quantum cryptography [4]. Hence quantum entanglement has been viewed as an essential resource for quantum information processing and all of these applications depend upon the strength of quantum entanglement. One of the most important aspects of quantum information processing is that information can be 'encoded' in non - local correlations (entanglement) between two separated particles.

The present work deals with the local copying and partial broadcasting of entangled pairs [5]. So we should have a preliminary idea about what we actually mean by partial broadcasting.

Let Alice and Bob share an inseparable (entangled) state whose density operator is given by ρ^{id}_{AB}.We will use two quantum copiers Q_1 and Q_2 (the density operator $\rho_{Q_1Q_2}$ describing the input state of two quantum copiers is separable) to locally copy A and B such that at the output level of A and B two other states C and D are produced respectively. As a result of this copying, we obtain, out of two systems A and B, four other systems described by a density operator ρ^{out}_{ABCD}. Now if we can see that the states ρ^{out}_{AD} and ρ^{out}_{BC} are inseparable while the states ρ^{out}_{AC} and ρ^{out}_{BD}, which are produced locally, are separable, then we can say that we have partially broadcasted (cloned, split) the entanglement (inseparability) that was present in the input state.

Our motivation for this present work is basically two fold :

(1) We discussed a protocol by which we can generate two bipartite entangled states between two parties Alice and Bob from a W-type state shared by Alice, Bob and Charlie without involving Charlie at all.For doing so, we have started with a three party entangled state and by Buzek - Hillery quantum cloners [6], we have locally copied the first two particles.Then without losing generality we have traced out the third party as well as the machine state, to produce four bipartite local and non - local output state . Then we

have considered the case $\beta = \gamma$ to find out the range of α^2 for which local output states are separable and the non local output states are inseparable.A pragmatic inference, if we can find the output states to be entangled , is that we can use them as channels to encode more information. Also it is seen that the third party in the three party entangled pair has no effect on the whole process.

(2) Keeping the range of α^2 fixed we find out that whether the mixed ness of the partially broadcasted subsystems obtained from the entangled input system does have any effect on their entanglement (inseparabilty) i.e. whether or not the amount of their entanglement have any relation with mixed ness. When entangled , we find out the amount of entanglement and check if there exists any relation between the amount of entanglement and mixed ness of the output states.

The paper can thus be summarized as follows:

In section 2, we have taken a non - maximal W - state (or W - type state) which is defined as $|\psi\rangle = \alpha|001\rangle_{123} + \beta|010\rangle_{123} + \gamma|100\rangle_{123}, \alpha^2 + \beta^2 + \gamma^2 = 1$. Then we locally copy first two qubits and then we trace out third qubit and machine state to obtain two bipartite entangled state . The logic behind taking W - type state and not W - state ($\alpha = \beta = \gamma = \frac{1}{\sqrt{3}}$) is that in case of W - state all the local and non - local bipartite output states become separable and there is nothing to prove further.

In section 3, by taking $\beta = \gamma$ we obtain the range of α^2 for which the local output states are separable and non local output states are inseparable.

In section 4, we made a comparative study of the mixed ness and amount of entanglement of the entangled and non - entangled bipartite states with the help of 'Linear Entropy' and 'Concurrence'.

In section 5, i.e. 'Conclusion', we have reviewed the previous sections and have given our concluding remark.

2 Analysis of non-maximal W state (or W type state):

For this we shall start with W type state which is given by-

$$|X\rangle = \alpha|001\rangle_{123} + \beta|010\rangle_{123} + \gamma|100\rangle_{123}, where, \alpha^2 + \beta^2 + \gamma^2 = 1 \tag{1}$$

. Here we do not consider the cases where, $\alpha = 1, \beta = 0, \gamma = 0$ or $\alpha = 0, \beta = 0, \gamma = 1$ or $\alpha = 0, \beta = 1, \gamma = 0$. We shall clone the first two bits of state (1) with the help of Buzek-Hillery quantum cloning machine, where transformations are given by the following:

$$|0\rangle \rightarrow \sqrt{\frac{2}{3}}|00\rangle|\uparrow\rangle + \sqrt{\frac{1}{6}}[|01\rangle + |10\rangle]|\downarrow\rangle \tag{2}$$

$$|1\rangle \rightarrow \sqrt{\frac{2}{3}}|11\rangle|\downarrow\rangle + \sqrt{\frac{1}{6}}[|01\rangle + |10\rangle]|\uparrow\rangle \tag{3}$$

Here $|\uparrow\rangle$ and $|\downarrow\rangle$ are machine states.After cloning the first two bits and tracing out the machine states as well as the third bit we get the density matrix of the resulting state as follows:

$$\begin{aligned}
\rho_{1245} = \alpha^2[\{\frac{2}{3}|00\rangle_{14}\langle 00| + \frac{1}{6}(|01\rangle_{14}\langle 01| + |01\rangle_{14}\langle 10| + |10\rangle_{14}\langle 01| + |10\rangle_{14}\langle 10|)\} \otimes \\
\{\frac{2}{3}|00\rangle_{25}\langle 00| + \frac{1}{6}(|01\rangle_{25}\langle 01| + |01\rangle_{25}\langle 10| + |10\rangle_{25}\langle 01| + |10\rangle_{25}\langle 10|)\}] + \\
\beta^2[\{\frac{2}{3}|00\rangle_{14}\langle 00| + \frac{1}{6}(|01\rangle_{14}\langle 01| + |01\rangle_{14}\langle 10| + |10\rangle_{14}\langle 01| + |10\rangle_{14}\langle 10|)\} \otimes \\
\{\frac{2}{3}|11\rangle_{25}\langle 11| + \frac{1}{6}(|01\rangle_{25}\langle 01| + |01\rangle_{25}\langle 10| + |10\rangle_{25}\langle 01| + |10\rangle_{25}\langle 10|)\}] + \\
\gamma^2[\{\frac{2}{3}|11\rangle_{14}\langle 11| + \frac{1}{6}(|01\rangle_{14}\langle 01| + |01\rangle_{14}\langle 10| + |10\rangle_{14}\langle 01| + |10\rangle_{14}\langle 10|)\} \otimes \\
\{\frac{2}{3}|00\rangle_{25}\langle 00| + \frac{1}{6}(|01\rangle_{25}\langle 01| + |01\rangle_{25}\langle 10| + |10\rangle_{25}\langle 01| + |10\rangle_{25}\langle 10|)\}] + \\
\beta\gamma[\{\frac{1}{3}(|00\rangle_{14}\langle 01| + |00\rangle_{14}\langle 10|) + \frac{1}{3}(|01\rangle_{14}\langle 11| + |10\rangle_{14}\langle 11|)\} \otimes \\
\{\frac{1}{3}(|11\rangle_{25}\langle 01| + |11\rangle_{25}\langle 10|) + \frac{1}{3}(|01\rangle_{25}\langle 00| + |10\rangle_{25}\langle 00|)\}] + \\
\gamma\beta[\{\frac{1}{3}(|11\rangle_{14}\langle 01| + |11\rangle_{14}\langle 10|) + \frac{1}{3}(|01\rangle_{14}\langle 00| + |10\rangle_{14}\langle 00|)\} \otimes \\
\{\frac{1}{3}(|00\rangle_{25}\langle 01| + |00\rangle_{25}\langle 10|) + \frac{1}{3}(|01\rangle_{25}\langle 11| + |10\rangle_{25}\langle 11|)\}]
\end{aligned} \tag{4}$$

Now from calculated ρ_{1245},we partially trace out the particle 4 and 2. The resulting state comes out as,

$$\begin{aligned}\rho_{15} = \alpha^2[&\frac{25}{36}|00\rangle_{15}\langle 00| + \frac{5}{36}|01\rangle_{15}\langle 01| + \frac{5}{36}|10\rangle_{15}\langle 10| + \frac{1}{36}|11\rangle_{15}\langle 11|] + \\ \beta^2[&\frac{5}{36}|00\rangle_{15}\langle 00| + \frac{25}{36}|01\rangle_{15}\langle 01| + \frac{1}{36}|10\rangle_{15}\langle 10| + \frac{5}{36}|11\rangle_{15}\langle 11|] + \\ \gamma^2[&\frac{25}{36}|10\rangle_{15}\langle 10| + \frac{5}{36}|11\rangle_{15}\langle 11| + \frac{5}{36}|00\rangle_{15}\langle 00| + \frac{1}{36}|01\rangle_{15}\langle 01|] + \\ &\beta\gamma[\frac{4}{9}|01\rangle_{15}\langle 10|] + \gamma\beta[\frac{4}{9}|10\rangle_{15}\langle 01|]\end{aligned} \tag{5}$$

Similarly tracing out partially the particle pairs (2,5),(1,4),and (1,5),we get the following resulting states respectively:

$$\begin{aligned}\rho_{14} = \alpha^2[&\frac{2}{3}|00\rangle_{14}\langle 00| + \frac{1}{6}(|01\rangle_{14}\langle 01| + |01\rangle_{14}\langle 10| + |10\rangle_{14}\langle 01| + |10\rangle_{14}\langle 10|)] + \\ \beta^2[&\frac{2}{3}|00\rangle_{14}\langle 00| + \frac{1}{6}(|01\rangle_{14}\langle 01| + |01\rangle_{14}\langle 10| + |10\rangle_{14}\langle 01| + |10\rangle_{14}\langle 10|)] + \\ \gamma^2[&\frac{2}{3}|11\rangle_{14}\langle 11| + \frac{1}{6}(|01\rangle_{14}\langle 01| + |01\rangle_{14}\langle 10| + |10\rangle_{14}\langle 01| + |10\rangle_{14}\langle 10|)]\end{aligned} \tag{6}$$

$$\begin{aligned}\rho_{25} = \alpha^2[&\frac{2}{3}|00\rangle_{25}\langle 00| + \frac{1}{6}(|01\rangle_{25}\langle 01| + |01\rangle_{25}\langle 10| + |10\rangle_{25}\langle 01| + |10\rangle_{25}\langle 10|)] + \\ \beta^2[&\frac{2}{3}|11\rangle_{25}\langle 11| + \frac{1}{6}(|01\rangle_{25}\langle 01| + |01\rangle_{25}\langle 10| + |10\rangle_{25}\langle 01| + |10\rangle_{25}\langle 10|)] + \\ \gamma^2[&\frac{2}{3}|00\rangle_{25}\langle 00| + \frac{1}{6}(|01\rangle_{25}\langle 01| + |01\rangle_{25}\langle 10| + |10\rangle_{25}\langle 01| + |10\rangle_{25}\langle 10|)]\end{aligned} \tag{7}$$

$$\begin{aligned}\rho_{42} = \alpha^2[&\frac{25}{36}|00\rangle_{42}\langle 00| + \frac{5}{36}|01\rangle_{42}\langle 01| + \frac{5}{36}|10\rangle_{42}\langle 10| + \frac{1}{36}|11\rangle_{42}\langle 11|] + \\ \beta^2[&\frac{5}{36}|00\rangle_{42}\langle 00| + \frac{25}{36}|01\rangle_{42}\langle 01| + \frac{1}{36}|10\rangle_{42}\langle 10| + \frac{5}{36}|11\rangle_{42}\langle 11|] + \\ \gamma^2[&\frac{5}{36}|00\rangle_{42}\langle 00| + \frac{1}{36}|01\rangle_{42}\langle 01| + \frac{25}{36}|10\rangle_{42}\langle 10| + \frac{5}{36}|11\rangle_{42}\langle 11|] + \\ &\beta\gamma[\frac{4}{9}|01\rangle_{42}\langle 10| + \frac{4}{9}|10\rangle_{42}\langle 01|]\end{aligned} \tag{8}$$

3 Analysis of the the Separability and Inseparability condition for the local and non local output states:

First of all in this section, we obtain the expression of the determinants W_3 and W_4 for local and non local output states by using Peres Horodecki criterion [7,8]. Then we consider the case when $(\beta = \gamma)$ and investigate the separability and inseparability criterion for local and nonlocal states. We also obtained the range of α^2 for which the local output states are separable and the nonlocal output states are inseparable.

We shall now try to find out the expression of W_3 and W_4 for the states ρ_{15}, ρ_{14},ρ_{25} and ρ_{42}, with the help of Peres-Horodecki Criterion.

Peres-Horodecki Theorem : The necessary and sufficient condition for the state ρ of two spin $\frac{1}{2}$ particles to be inseparable is that at least one of the eigen values of the partially transposed operator defined as $\rho^T_{m\mu,n\nu} = \rho_{m\mu,n\nu}$, is negative. This is equivalent to the condition that at least one of the two determinants

$$W_3 = \begin{vmatrix} \rho_{00,00} & \rho_{01,00} & \rho_{00,10} \\ \rho_{00,01} & \rho_{01,01} & \rho_{00,11} \\ \rho_{10,00} & \rho_{11,00} & \rho_{10,10} \end{vmatrix} \text{ and } W_4 = \begin{vmatrix} \rho_{00,00} & \rho_{01,00} & \rho_{00,10} & \rho_{01,10} \\ \rho_{00,01} & \rho_{01,01} & \rho_{00,11} & \rho_{01,11} \\ \rho_{10,00} & \rho_{11,00} & \rho_{10,10} & \rho_{11,10} \\ \rho_{10,01} & \rho_{11,01} & \rho_{10,11} & \rho_{11,11} \end{vmatrix}$$

is negative.

The values of W_3 and W_4 for the different states ρ_{15},ρ_{14},ρ_{25} an ρ_{42} are as follows:

For non local output states $\rho_{15} = \rho_{42}$:

$$W_3 = \begin{vmatrix} \frac{25}{36}\alpha^2 + \frac{5}{36}\beta^2 + \frac{5}{36}\gamma^2 & 0 & 0 \\ 0 & \frac{5}{36}\alpha^2 + \frac{25}{36}\beta^2 + \frac{1}{36}\gamma^2 & 0 \\ 0 & 0 & \frac{5}{36}\alpha^2 + \frac{1}{36}\beta^2 + \frac{25}{36}\gamma^2 \end{vmatrix}$$

and

$$W_4 = \begin{vmatrix} \frac{25}{36}\alpha^2 + \frac{5}{36}\beta^2 + \frac{5}{36}\gamma^2 & 0 & 0 & \frac{4}{9}\beta\gamma \\ 0 & \frac{5}{36}\alpha^2 + \frac{25}{36}\beta^2 + \frac{1}{36}\gamma^2 & 0 & 0 \\ 0 & 0 & \frac{5}{36}\alpha^2 + \frac{1}{36}\beta^2 + \frac{25}{36}\gamma^2 & 0 \\ \frac{4}{9}\beta\gamma & 0 & 0 & \frac{1}{36}\alpha^2 + \frac{5}{36}\beta^2 + \frac{5}{36}\gamma^2 \end{vmatrix}$$

For local output states ρ_{14} and ρ_{25} :

For ρ_{14}:

$$W_3 = \begin{vmatrix} \frac{2}{3}(\alpha^2+\beta^2) & 0 & 0 \\ 0 & \frac{1}{6} & 0 \\ 0 & 0 & \frac{1}{6} \end{vmatrix}$$

$$\text{and } W_4 = \begin{vmatrix} \frac{2}{3}(\alpha^2+\beta^2) & 0 & 0 & \frac{1}{6} \\ 0 & \frac{1}{6} & 0 & 0 \\ 0 & 0 & \frac{1}{6} & 0 \\ \frac{1}{6} & 0 & 0 & \frac{2}{3}\gamma^2 \end{vmatrix}$$

For ρ_{25}:

$$W_3 = \begin{vmatrix} \frac{2}{3}(\alpha^2+\gamma^2) & 0 & 0 \\ 0 & \frac{1}{6} & 0 \\ 0 & 0 & \frac{1}{6} \end{vmatrix}$$

$$\text{and } W_4 = \begin{vmatrix} \frac{2}{3}(\alpha^2+\gamma^2) & 0 & 0 & \frac{1}{6} \\ 0 & \frac{1}{6} & 0 & 0 \\ 0 & 0 & \frac{1}{6} & 0 \\ \frac{1}{6} & 0 & 0 & \frac{2}{3}\beta^2 \end{vmatrix}$$

Now if we put $\beta = \gamma$, then the density matrices representing the subsystems as given by equations (5), (6), (7), (8) becomes:

$$\begin{aligned} \rho_{14} = \rho_{25} = & \frac{1}{3}(1+\alpha^2)|00\rangle\langle 00| + \frac{1}{6}(|01\rangle\langle 01| + |10\rangle\langle 10| \\ & +|01\rangle\langle 10| + |10\rangle\langle 01|) + \frac{1}{3}(1-\alpha^2)|11\rangle\langle 11| \end{aligned} \quad (9)$$

and

$$\rho_{15} = \rho_{42} = \frac{5}{36}(1+4\alpha^2)|00\rangle\langle 00| + \frac{1}{36}(13-8\alpha^2)(|01\rangle\langle 01| + |10\rangle\langle 10|)$$
$$+\frac{1}{36}(8-8\alpha^2)(|01\rangle\langle 10| + |10\rangle\langle 01|) + \frac{1}{36}(5-4\alpha^2)|11\rangle\langle 11| \quad (10)$$

Separability and Inseparability criterion for the subsystem ρ_{14} and ρ_{25} :

For these subsystems, $W_3 > 0$ and $W_4 = \frac{1}{6^4}(3-4\alpha^4)$.
Now $W_4 \geq 0$ if $\rho_{14}$and ρ_{25} are separable. A simple calculation reveal that $\rho_{14}$and ρ_{25} are separable when input parameter α^2 lies in the range (0,.86].

Separability and Inseparability criterion for the subsystem ρ_{15} and ρ_{42}:

For these subsystems, $W_3 > 0$ and $W_4 = \frac{1}{36^4}(5(1+4\alpha^2)(5-4\alpha^2) - (8-8\alpha^2)^2)$. For inseparability of $\rho_{15}$and ρ_{42} we must have, $W_4 \leq 0$. A simple calculation reveal that $\rho_{15}$and ρ_{42} are inseparable when input parameter α^2 lies in the range (0,.22).

In the common interval (0, 0.22) for α^2 although ρ_{14} and ρ_{25} are separable, ρ_{15} and ρ_{42} are entangled.

4 A comparative study of the Entanglement and Mixed ness of the local and non local output states:

In this section we have made a comparative study of the mixed ness and entanglement of the local and non local subsystems when $\alpha^2 \in (0, 0.22)$. For that reason first of all we find out the concurrence and linear entropy of the subsystems to quantify the amount of entanglement and mixed ness in them respectively. To find out the amount of entanglement we generally use Wootters formula of Concurrence.
Concurrence or Entanglement of Formation: Wootters [9,10] gave out, for the

mixed state $\hat{\rho}$ of two qubits, the concurrence is

$$C = max(\lambda_1 - \lambda_2 - \lambda_3 - \lambda_4, 0) \tag{11}$$

where the λ_i, in decreasing order, are the square roots of the eigen values of the matrix $\rho^{\frac{1}{2}}(\sigma_y \otimes \sigma_y)\rho^*(\sigma_y \otimes \sigma_y)\rho^{\frac{1}{2}}$ denotes the complex conjugation of ρ in the computational basis $\{|00\rangle, |01\rangle, |10\rangle, |11\rangle\}$ and σ_y is the Pauli operator. The entanglement of formation E_F can then be expressed as a function of C, namely

$$E_F = -\frac{1+\sqrt{1-C^2}}{2}\log_2\frac{1+\sqrt{1-C^2}}{2} - \frac{1-\sqrt{1-C^2}}{2}\log_2\frac{1-\sqrt{1-C^2}}{2} \tag{12}$$

Concurrence for the subsystems ρ_{14} and ρ_{25}:

Here we investigate the amount of entanglement present in these subsystems when the input parameter α^2 lies in the range (0,.22).
$C(\rho_{14}, \rho_{25}) = 2max(\frac{1}{6} - \frac{1}{6}(\sqrt{4-4\alpha^4}), 0) = 0$ in $0 < \alpha^2 < 0.22$.

Concurrence for the subsystems ρ_{15} and ρ_{42}:

Here also we investigate the amount of entanglement present in these subsystems when the input parameter α^2 lies in the range (0,.22).
$C(\rho_{15}, \rho_{42}) = 2max(\frac{8-8\alpha^2}{36} - \frac{1}{36}(\sqrt{5(1+4\alpha^2)(5-4\alpha^2)}), 0)$ Now for $0 < \alpha^2 < 0.22$, the concurrence $C(\rho_{15}, \rho_{42})$ of the subsystems lies in the range (.001,.17).
Linear Entropy for the subsystems ρ_{14} and ρ_{25}:

The expression for the state dependent Linear Entropy is defined as:
$S_L(\rho) = \frac{4}{3}(1 - Tr(\rho^2))$ [11]
Here we investigate the amount of mixed ness in the subsystems ρ_{14} and ρ_{25} when the input parameter α^2 lies in the range (0,.22).
The linear entropy for these subsystems is given by,
$S_L(\rho_{14}) = S_L(\rho_{25}) = \frac{8}{27}(3 - \alpha^4)$. Now when $\alpha^2 \in (0, .22)$, then $S_L(\rho_{14}, \rho_{25}) \in (.87, .89)$.

Linear Entropy for the subsystems ρ_{15} and ρ_{42}:

The Linear Entropy in this case is: $S_L(\rho_{15}) = S_L(\rho_{42}) = \frac{4}{3}[-\frac{1}{324}(168\alpha^4 - 12\alpha^2 + 129) + 1]$
We now investigate the amount of mixed ness in the subsystems ρ_{15} and ρ_{42} when the input parameter α^2 lies in the range (0,.22).
Now when $\alpha^2 \in (0, .22)$, then $S_L(\rho_{15}, \rho_{42}) \in (.77, .81)$.

From the above calculations of the linear entropy it is clear that the mixed ness of the non-local outputs are less than the local output states for the same values of α^2. This opens up the possibility of extracting pure entanglement efficiently from these two partially entangled state between Alice and Bob.
Since it is evident from the previous section that the local output states are separable and non local output states are inseparable when $\alpha^2 \in (0, 0.22)$, it remains interesting to have a comparative study of the mixed ness and entanglement of the subsystem. Here in the following table we show the mixed ness and the amount of entanglement of the subsystem when the input probability α^2 lies in the range (0,0.22).

TABLE 2 ($\alpha^2 \in (0, 0.22)$):

Subsystems	Linear Entropy	Concurrence
ρ_{15} and ρ_{42}	(.77,.81)	(.001,.17)
ρ_{14} and ρ_{25}	(.87,.89)	0

5 Conclusions

In summary we can say that in this protocol we are able to generate two bipartite entangled state between two friends Alice and Bob from a tripartite entangled state initially shared between Alice, Bob and Carol. We also find out under what conditions the non-local

output states are inseparable and local output states are separable. We also considered the case when $\beta = \gamma$, and also made a comparative study between the amount of entanglement and mixed ness of the local and non local subsystems. The interesting result that we obtain is that the local outputs , although separable , have a higher degree of mixed ness than the non-local outputs. While comparing the mixed ness and entanglement of the non-local outputs , we have found out that the mixed ness and concurrence share a positive correlation. Thus if the mixed ness of the non-local outputs increases, their amount of entanglement also increases and consequently we can encode more information in these non-local outputs. So, we can say that mixed ness has a positive impact on data encoding.

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7 References

[1] C.H.Bennett, G.Brassard, C.Crepeau, R.Jozsa, A.Peres, W.K.Wootters, Phys Rev Lett. 70 (1993), 1895

[2]C.H.Bennett, S.J.Wiesner, Phys Rev Lett, 69 (1992), 2881

[3] P.W.Shor, Phys Rev A 52 (1995) 2493

[4] T. Jennewein, C.Simon, G. Weihs, H.Weinfurter and A.Zeilinger, Phys Rev Lett, 84 (200) 4729

[5]V.Buzek, V.Vedral, M.B.Plenio, P.L.Knight, M.Hillery, quant-ph/9701028v1,23Jan, 1997

[6] V.Buzek, M.Hillery, quant-ph/9607018v1, 20 Jul, 1996.

[7]M.Horodecki, P.Horodecki, R.Horodecki, PLA 223, 1 (1996)

[8]A.Peres, PRL 77, 1413, 1996

[9] Hill, S and W.K.Wootters,PRL 78, (5022),1997

[10] Wootters, W.K, PRL 80, 2245, 1998

[11] Quantum Computation and Quantum Information, M.A. Nielsen and I.L.Chuang, Cambridge University Press.